Introduction to Tone-Clock Theory

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Jenny McLeod  *Tone Clock Piece No 1 (1988)*

Michael Houstoun (pf)
*From ‘24 Tone Clocks’ (Rattle Records)*
About Tone-Clock Theory
Tone-clock theory ‘founders’

- **Peter Schat** (Dutch composer 1935–2003)
  - ‘Toonklok’: theory of chromatic triads and special transpositional processes

- **Jenny McLeod** (NZ composer 1941–)
  - developed Schat’s ideas by expanding them to other cardinalities (using Forte’s pc-set theory)
About Tone-Clock Theory (TCT)

- **TONE-CLOCK COMPOSITION**
  - predicated on the formation of pitch structure in which intervallic makeup is highly delimited, while pitch-class variety is high

- **TONE-CLOCK ANALYSIS** is less developed and more speculative
  - TCT could be used support claims of familial relationships between pc-sets from the POV of intervallic genera, and show how transpositional operations can be considered a musical object in itself (a form of deep structure)
Tone clock theory in context
Tone-clock theory in context

- Interested in:
  - AGGREGATE FORMATION
  - INTERVALLIC ECONOMY AND ‘FLAVOUR’
  - SYMMETRY
  - ‘TRANSPOSITIONAL HARMONY’

- Ties together:
  - the atonal & (some) serial works of the Second Viennese School (esp. Webern)
  - Transposition techniques & symmetries of Franco-Russian school (e.g. planing, whole-tone, octatonic)
Precursors

- WEBERN’s ‘serial derivation’
- HAUER’s ‘trope’ theory
- Pre-serial atonality of SCHOENBERG
- ‘Chromatic planing’ of DEBUSSY
- Cyclicity and symmetry in music of BARTÓK and MESSIAEN
- BOULEZ’s ‘frequency multiplication’
- FORTE’s pitch-class set theory
Tone-clock theory

- TCT inherits from pc-set:
  - set-class as a free (unordered) entity, prime form

- TCT does not consider:
  - interval vector, Z-relations, similarity indices

- TCT extends pc-set theory:
  - by examining the patterns that occur **WHEN YOU TRANSPOSE AND/OR INVERT SET-CLASSES BY OTHER SET-CLASSES**
  - ...and operations on a single set-class that **FORM THE AGGREGATE**
Ordering

- Strict order is unimportant
  - takes a general ‘unordered’ (atonal) approach from free atonality
  - but pc-sets must be composed ‘proximately’ (nearby), so we can hear the quality of the set-class, but the specific order is not important

  (issue of segmentation for analysts)
A few key pc-set principles to recap
Recap of pc-set theory

- Pc-sets have a **prime form** (‘most compact form’)
- Set-classes are invariant under **transposition & inversion**
- Distribution of **set-classes by cardinality**:

<table>
<thead>
<tr>
<th>$k$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td># of patterns</td>
<td>1</td>
<td>6</td>
<td>12</td>
<td>29</td>
<td>38</td>
<td>50</td>
<td>38</td>
<td>29</td>
<td>12</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 2: Number of patterns of $k$-Chords in 12-tone music with regard to $v_n^{(E)}$.

Table reproduced from Harald Fripertinger and Graz Voitsberg, ‘Enumeration in Musical Theory’ (Institut Für Elektronische Musik (IEM), 1992).
Mathematical terminology

- A set-class is a ‘combinatorial bracelet’
  - a circle of 12 beads, where each bead is either coloured or blank
  - bracelet retains identity under rotation or reversal

See https://en.wikipedia.org/wiki/Necklace_%28combinatorics%29#Number_of_bracelets
Tone-clock terminology
Intervallic Prime Form (IPF)
The Intervalaric Prime Form (IPF)

- Same as Forte’s ‘prime form’
  - but notated using what I call the ‘short ic-form’ as the ‘name’, rather than a zero-based pc-set or Forte catalogue number

\[
\begin{align*}
2 & 5 \\
\end{align*}
\]

\[= 2-5\]
The Intervallic Prime Form (IPF)

- Triads take a hyphen in the middle
  - e.g. \{C₄, C♯, E₄\} = 1-3
  - ... but \{C₄, C♯, E₄, F₄\} = 131

- NB: IPFs can take multiple names, especially if there are intervallically simpler forms
  - e.g. 2–5 can be also spelled as 5–5
Hours

- The trichords (called ‘triads’ in TCT) form a crucial part of familial relationships between different IPFs
  - Contain 1 or 2 generating interval-classes that create the fundamental harmonic ‘flavour’ of a tone-clock work

- As there are 12 triads, these are called the 12 ‘hours’ of the tone clock
The ‘hours’

- Each hour is given a roman numeral:

<table>
<thead>
<tr>
<th>Roman Numeral</th>
<th>Numerical Equivalent</th>
</tr>
</thead>
<tbody>
<tr>
<td>I: 1-1</td>
<td>VII: 2-3</td>
</tr>
<tr>
<td>II: 1-2</td>
<td>VIII: 2-4</td>
</tr>
<tr>
<td>III: 1-3</td>
<td>IX: 2-5/5-5</td>
</tr>
<tr>
<td>IV: 1-4</td>
<td>X: 3-3</td>
</tr>
<tr>
<td>V: 1-5</td>
<td>XI: 3-4</td>
</tr>
<tr>
<td>VI: 2-2</td>
<td>XII: 4-4</td>
</tr>
</tbody>
</table>
The twelve ‘hours’ (presented on C)
The harmonic ‘flavours’ of the hours (cf. pc-set genera)
Table 10
The Pitch-Class Set Genera

<table>
<thead>
<tr>
<th>Genus</th>
<th>Type</th>
<th>Progenitor(s)</th>
<th>Counts (#3/#4/#5/#6)*</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt;1</td>
<td>atonal</td>
<td>3-5</td>
<td>1/9/24/29</td>
</tr>
<tr>
<td>&gt;2</td>
<td>whole-tone</td>
<td>3-8</td>
<td>1/9/24/30</td>
</tr>
<tr>
<td>&gt;3</td>
<td>diminished</td>
<td>3-10</td>
<td>1/5/16/21</td>
</tr>
<tr>
<td>4</td>
<td>augmented</td>
<td>3-12</td>
<td>1/2/8/9</td>
</tr>
<tr>
<td>&gt;5</td>
<td>chroma</td>
<td>3-1 &amp; 3-2</td>
<td>2/2/10/15</td>
</tr>
<tr>
<td>&gt;6</td>
<td>semichroma</td>
<td>3-2 &amp; 3-3</td>
<td>2/3/16/24</td>
</tr>
<tr>
<td>7</td>
<td>chroma-dia</td>
<td>3-2 &amp; 3-7</td>
<td>2/3/15/25</td>
</tr>
<tr>
<td>&gt;8</td>
<td>atonal</td>
<td>3-3 &amp; 3-4</td>
<td>2/3/15/21</td>
</tr>
<tr>
<td>&gt;9</td>
<td>atonal-tonal</td>
<td>3-3 &amp; 3-11</td>
<td>2/3/15/21</td>
</tr>
<tr>
<td>&gt;10</td>
<td>atonal-tonal</td>
<td>3-4 &amp; 3-11</td>
<td>2/3/15/21</td>
</tr>
<tr>
<td>&gt;11</td>
<td>dia</td>
<td>3-7 &amp; 3-9</td>
<td>2/2/10/15</td>
</tr>
<tr>
<td>&gt;12</td>
<td>dia-tonal</td>
<td>3-7 &amp; 3-11</td>
<td>2/3/16/24</td>
</tr>
</tbody>
</table>

*Each pair of z-related hexachords receives one count.
Hour symmetry
Symmetrical vs asymmetrical hours

- Symmetrical hours are constructed from **ONE INTERVAL CLASS ONLY**
  - 1-1, 2-2, 3-3, 4-4
  - = I, VI, X, XII

- Asymmetrical hours are constructed from **TWO INTERVAL CLASSES**
  - 1-2, 1-3, 1-4, 1-5, 2-3, 2-4, 2-5, 3-4
  - = II, III, IV, V, VII, VIII, IX, XI
Minor & major forms
Minor & major forms

- Assymetrical hours can be written in ‘minor’ or ‘major’ form:

  **MINOR FORM**
  - asymmetrical hour with smallest interval first
  - indicated with lower-case m — e.g. **IVm** (1-4)

  ![Minor Form Diagram]

  **MAJOR FORM**
  - asymmetrical hour with largest interval first
  - indicated with capital M — e.g. **IVM** (4-1)

  ![Major Form Diagram]
Hour-groups
If an IPFs larger than a triad is uni- or bi-intervallic, it can be expressed as an ‘hour group’

- e.g. 131 is generated from III (1-3)
- Label: $\text{III}^4$ (‘third-hour tetrad’)
- Can also take minor and major form, which will be different set-classes
  - $131 = \text{III}m^4$ \hspace{0.5cm} (0145 = 4-7)
  - $313 = \text{III}M^4$ \hspace{0.5cm} (0347 = 4-17)
Larger hour groups

- **OEDIPUS GROUPS**
  - Groups larger than tetrads, in which two intervals alternate
    - e.g. 1313 = lllm⁵ (‘third-hour Oedipus pentad’) (01458 = 5-21)

- **SYMMETRICAL PENTADS** (SPs)
  - Five-note groups of two intervals that are symmetrical
    - e.g. 1331 = SPIllm (01478 = 5-22)

- **GEMINI GROUPS** (heptads): 343-343
OTHER hour groups

- **OEDIPUS GROUPS**
  - Two ics alternate (i.e. interval cycle subset) for pentads or larger
  - Indicate using a superscript number showing the cardinality
    - e.g. $1313 = \text{III}^5$; hexatonic scale $= 13131 = 31313 = \text{III}^6$

- **SYMMETRICAL PENTADS** (SPs)
  - Pentads of two ics, symmetrically arrayed (i.e. palindromic)
  - Indicate by writing ‘SP’
    - e.g. $2332 = \text{SPVII}m$ (NB: can also be written as $4334 = \text{SPXIM}$)
Multiple-nature hour groups

\[252 = IX^4\]

\[232 = VII^4\]
### Multiple-Nature Hour-Groups

<table>
<thead>
<tr>
<th>Fig. 1.1</th>
<th>I</th>
<th>II</th>
<th>IX</th>
<th>VII</th>
<th>XI</th>
<th>IV</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4.23</td>
<td></td>
<td>IX^4 (=252)</td>
<td>= 232</td>
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<tr>
<td></td>
<td>4.26</td>
<td></td>
<td>323</td>
<td>= 343</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>4.34</td>
<td></td>
<td>434</td>
<td>= 414</td>
<td></td>
<td></td>
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<td></td>
<td>5.34</td>
<td></td>
<td>2332</td>
<td>= 4334</td>
<td></td>
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<tr>
<td></td>
<td>5.35</td>
<td></td>
<td>IX^5 (=5225)</td>
<td>= 3223 = 2323</td>
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<tr>
<td></td>
<td>5.17 [Z37]</td>
<td></td>
<td>3443</td>
<td>= 1441</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>6.32</td>
<td></td>
<td>IX^6 (=25225)</td>
<td>= 32323</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.26 [Z48]</td>
<td></td>
<td>34343</td>
<td>= 41414</td>
<td></td>
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<tr>
<td></td>
<td>7.17 [Z37]</td>
<td></td>
<td>323-323</td>
<td>= 414-414</td>
<td>= 313-313</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.32</td>
<td>121221</td>
<td></td>
<td>343434</td>
<td>= 131-131</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.35</td>
<td>212-212</td>
<td>IX^7 (=252-252)</td>
<td>= 343434</td>
<td>= 343-343</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.22</td>
<td></td>
<td>434-434</td>
<td>= 131-131</td>
<td></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>8.72</td>
<td></td>
<td>3434343</td>
<td>= 1331331</td>
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<tr>
<td></td>
<td>8.20</td>
<td>1441441</td>
<td>= 3113113</td>
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<td></td>
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<tr>
<td></td>
<td>8.23</td>
<td>121-121-121 = 212-2112</td>
<td></td>
<td>434-4334</td>
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<td></td>
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<tr>
<td></td>
<td>8.24</td>
<td>1221221</td>
<td>IX^8 (=5225225)</td>
<td>= 4343434</td>
<td>= 4141414</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>8.14</td>
<td>2112112</td>
<td>= 23252352</td>
<td></td>
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<td></td>
<td>8.22</td>
<td>2323-323</td>
<td>= 1414-414</td>
<td></td>
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<td></td>
<td>9.64</td>
<td>323-2332</td>
<td>= 414-441</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>9.11</td>
<td>121-121-121</td>
<td>= 323-3232</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.12</td>
<td><strong>121-121-121</strong></td>
<td>= <strong>232-323-323</strong></td>
<td>= 1441441</td>
<td>= <strong>1441441</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.9</td>
<td>IX^9 (=252-25225)</td>
<td>= 2323-3232</td>
<td>= 1414-4141</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9.24</td>
<td>2112112</td>
<td>= 232323232</td>
<td>= 141144141</td>
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<td></td>
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</tr>
<tr>
<td></td>
<td>9.34</td>
<td>23232332</td>
<td><strong>2343443443</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.1</td>
<td>1212112</td>
<td>= 23232332</td>
<td>414-414-414</td>
<td>= 131-131-131</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.5</td>
<td>10.5</td>
<td>IX^10 (=252-252-252)</td>
<td>= 32323-3232</td>
<td>= 434-434-434</td>
<td>= 141414141</td>
<td>= 313-313-313</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>11</td>
<td>IX^11 (=5225252525)</td>
<td>= 32323-32323</td>
<td>= 3434-433434</td>
<td>1441441441</td>
<td>1333131313</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>12</td>
<td>IX^12 (==252-252-252-252)</td>
<td>= 4334-434-4334</td>
<td><strong>=414-414-414-414</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Generative structures of hour groups

1-3 (=3-1)

131

1313 (=3131)

13131

131313 (=313131)

1331

3113

313

31313

3113
Steering, steering groups & fields
Steering

- Transposition of one IPF by another (the steering group)
  - Self-steering is possible
  - Complete set of transpositions creates a ‘field’
  - Because IPFs are set-classes, inversions are also acceptable
Example

VIII steered by VI
Steering group

- Steering group = ‘deep structure’
  - Creates internal tension within IPF juxtapositions (cf. triadic harmony)
  - Analytical & perceptual assumption: we can ‘hear’ the influence of the steering IPF on the steered IPF
Self steering of 2-5
Tone-clock steering
Tone-clock steering

- Schat’s ‘revelation’ was that all 12 hours (except one!) can be **TRANSPOSED AND/OR INVERTED** SO THAT ALL 12 PC ARE GENERATED ONCE AND ONCE ONLY

- This is called ‘**TONE-CLOCK STEERING**’ and was extrapolated to as many of the 223 set-classes (IPFs) as possible by McLeod
  - The 11 possible triadic tone-clock steerings are called ‘**TRIADIC TONE-CLOCK TONALITIES**’
Tone-clock steering

- The symmetrical hours can be steered with transposition only
  - Let’s take an easy example: 4-4 (048)
Tone-clock steering of XII
Tone-clock steering

- The asymmetrical hours have to be steered with transposition **AND** inversion
  - e.g. 1-4 (015)
Tone-clock steering of 1-4
‘The Zodiac of the Hours’

after Peter Schat
Tessellation/tiling

- Tone-clock steering, in geometrical terms, is a form of pitch-class 'TESSELLATION' or 'TILING'
  - On the chromatic circle, this means a shape that is put through operations that maintain the shape (bracelets), so that every member of the chromatic group is created once & once only
    - **ROTATION** (transposition)
    - **REFLECTION** (inversion)
Tessellation/tiling

- c.f. works of M C Escher
  - Tone-clock steering shares the same ‘magic’ as these geometrical feats (cf. Messiaen’s ‘charm of impossibilities’)
Appendix IX:
12-Note Triadic Tone-Clock Tonalities (in Stave Notation)

NB: The three notes within each triad may come in any order, and the four triads in each field may also come in any order.
Transpositions refer to the number of possible transpositions before the same triad pitch-class content is repeated (regardless of note-order or triad-order, and regardless of which notes happen to be the steering-notes).

FIRST HOUR
I/IX 4
3 transpositions

SECOND HOUR
II/VIII 4
6 transpositions

THIRD HOUR
III/V 4
6 transpositions

FOURTH HOUR
III/IX 4
12 transpositions

FIFTH HOUR
IV/VIII 4
6 transpositions

SIXTH HOUR
IVm/IX 4
3 transpositions

SEVENTH HOUR
V/IIIM 4
12 transpositions

EIGHTH HOUR
VI/V 4
6 transpositions

NINTH HOUR
VI/IX 4
3 transpositions

ELEVENTH HOUR
VIII/IIM 4
12 transpositions

XII/IIM 4
0 transpositions

TENTIETH HOUR
XII/IX 4
0 transpositions

Also IX/IX 4
Analysis of Tone-Clock works
Analysis of Tone-Clock works

- EXAMPLE
  - JENNY MCLEOD • Tone Clock Piece I
JENNY MCLEOD • *Tone Clock Piece I*

- How is the opening FIELD constructed?
Tone-clock steering of IX
Development techniques in TCT
Development of auxiliary IPFs
IPF superimposition

mf (firm) cresc. 3

IX steered by 2

Aggregate = D♭ DIA
IPF superimposition

ff (with strength) appassionato

IX steered by 1

Aggregate = chromatic clusters
Anchor forms & reverse steering
Anchor form & reverse steering
Reverse Steering & Anchor Form

- **REVERSE STEERING**
  - Roles of the ‘steering IPF’ and ‘steered IPF’ swap

- **ANCHOR FORM**
  - A field comprising (usually) three symmetrical tetrads made from two different IPFs
    - a symmetrical IPF forms the ‘anchor’, and the other IPF is arrayed symmetrically around it
Questions & future work

- Perceptual ontology of steering group & reverse steering
- More fleshed-out analytical insights into specific compositional treatments & developments of fields (e.g. superposition)
- Analytical inapplicability to more general, multi-intervallic atonal structures
- Relationship to pc-set genera & pc-set similarity metrics
Readings & tools

- McLeod, J, ‘Tone Clock Theory Expanded: Chromatic Maps I & II’, unpublished manuscript
- Ibarz, Erik Fernandez, ‘Peter Schat’s Tone Clock: The Steering Function and Pitch-Class Set Transformation in Genen’ (University of Ottawa, 2015) <http://www.ruor.uottawa.ca/handle/10393/32397>
- Schat, Peter, *Tone Clock* (Routledge, 2012)
- **IPF FINDER**: [http://www.michaelnorris.info/theory/ipffinder](http://www.michaelnorris.info/theory/ipffinder)